Automated neuron tracing using probability hypothesis density filtering

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Supplementary Information

Algorithm 1 Neuron tracing

$k = 0$ ▷ Initialize

$\{\omega^n_{0|0}, \mathrm{x}^n_{0|0}\}_{n=1}^{\rho N_0}$ ▷ Initial particle and observation set

$\{\hat{\mathrm{x}}_{0,i}\}_{i=1}^{N_0}$ ▷ Initial estimate

repeat

    $k = k + 1$

    $\mathrm{p}^n_i \sim h(\mathrm{p}|\hat{\mathrm{x}}_{k-1,i}) \quad n \in [1, \rho N_{k-1}]$ ▷ Draw observation particles

    $\mathrm{p}^n_{i,j} \in \mathcal{C}_j, \quad j \in [1, M_k]\,, \quad n \in [1, |\mathcal{C}_j|]$ ▷ Cluster observation particles

    $\mathrm{z}_{k,j} = \left[\mathrm{p}^{\hat{n}}_{i,j}, \tau(\mathrm{p}^{\hat{n}}_{i,j})\right]$ ▷ Select representative sample

    $\mathrm{Z}_k = \{\mathrm{z}_{k,j}, \ldots, \mathrm{z}_{k,M_k}\}$ ▷ Construct observations

    $\{\omega^n_{k|k}, \mathrm{x}^n_{k|k}\}_{n=1}^{\rho N_k}, \nu_k, \{\hat{\mathrm{x}}_{k,i}\}_{i=1}^{N_k} \leftarrow$ SMC-PHD$(\{\omega^n_{k-1|k-1}, \mathrm{x}^n_{k-1|k-1}\}_{n=1}^{\rho N_{k-1}}, \mathrm{Z}_k)$ ▷ Algorithm 2

until $[\nu_k] = 0$ ▷ $[\cdot] \equiv$ nearest integer

Algorithm 2 SMC-PHD filtering

Input: $\{(\omega^n_{k-1|k-1}, \mathrm{x}^n_{k-1|k-1})\}^{\rho N_{k-1}}_{n=1}, \{\mathrm{z}_{k,j}\}^{M_k}_{j=1}$ $\triangleright$ $D_{k-1}(\mathrm{x})$ approx. observation Z_k

for $n = 1, \ldots, \rho N_{k-1}$ **do**

  for $m = 1, \ldots, \eta$ **do**

    $i = (n-1)\eta + m$

    Draw: $\mathrm{x}_{k|k-1,\mathrm{p}} \sim \pi_{k|k-1}(\mathrm{x}|\mathrm{x}^n_{k-1|k-1}) \rightarrow \mathrm{x}^i_{k|k-1,\mathrm{p}}$ $\triangleright$ Persistent object particles

    Compute: $\omega^i_{k|k-1,\mathrm{p}} = p_S \frac{1}{\eta} \omega^n_{k-1|k-1}$

    Draw: $\mathrm{x}_{k|k-1,\mathrm{s}} \sim \beta_{k|k-1}(\mathrm{x}|\mathrm{x}^n_{k-1|k-1}) \rightarrow \mathrm{x}^i_{k|k-1,\mathrm{s}}$ $\triangleright$ Spawning object particles

    Compute: $\omega^i_{k|k-1,\mathrm{s}} = p_S \frac{1}{\eta} \omega^n_{k-1|k-1}$

  end for

end for

$\{(\omega^n_{k|k-1}, \mathrm{x}^n_{k|k-1})\}^{S_k}_{n=1} = \{(\omega^n_{k|k-1,\mathrm{p}}, \mathrm{x}^n_{k|k-1,\mathrm{p}})\}^{\rho\eta N_{k-1}}_{n=1} \cup \{(\omega^n_{k|k-1,\mathrm{s}}, \mathrm{x}^n_{k|k-1,\mathrm{s}})\}^{\rho\eta N_{k-1}}_{n=1}$ $\triangleright$ Union of particle sets

for $n = 1, \ldots, S_k$ **do**

  Update: $\omega^n_{k|k} = (1-p_D)\omega^n_{k|k-1} + \sum\limits_{\mathrm{z} \in \mathrm{Z}_k} \frac{p_D g_k(\mathrm{z}|\mathrm{x}^n_{k|k-1})\omega^n_{k|k-1}}{C_k(\mathrm{z}) + \sum^{S_k}_{n=1} p_D g_k(\mathrm{z}|\mathrm{x}^n_{k|k-1})\omega^n_{k|k-1}}$

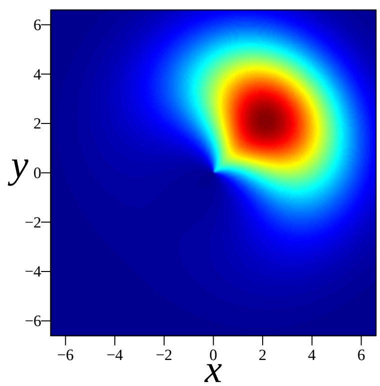
end for

$\nu_k = \sum\limits^{S_k}_{n=1} \omega^n_{k|k}$ $\triangleright$ Cardinality calculation

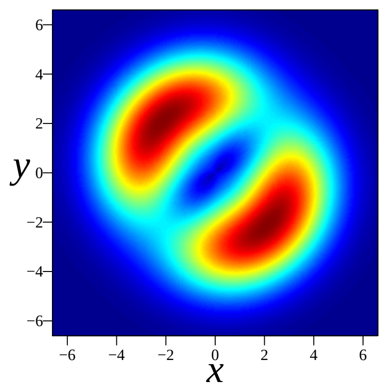
Estimate: $\hat{\mathrm{x}}_{k,i} \leftarrow \{\omega^n_{k|k}, \mathrm{x}^n_{k|k-1}\}^{S_k}_{n=1}$ $\triangleright$ Mean-shift clustering

Resample: $N_k = [\nu_k], \{\omega^n_{k|k}, \mathrm{x}^n_{k|k-1}\}^{S_k}_{n=1} \rightarrow \{\omega^n_{k|k}, \mathrm{x}^n_{k|k}\}^{\rho N_k}_{n=1}, \omega^n_{k|k} = \nu_k/(\rho N_k)$

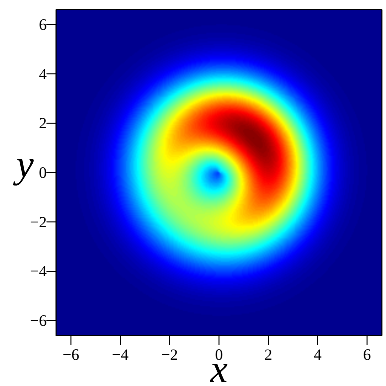
$\triangleright$ Systematic resampling with ρ particles per object



(A) $\pi_{k|k-1}(\mathrm{x}|\mathrm{x}')$

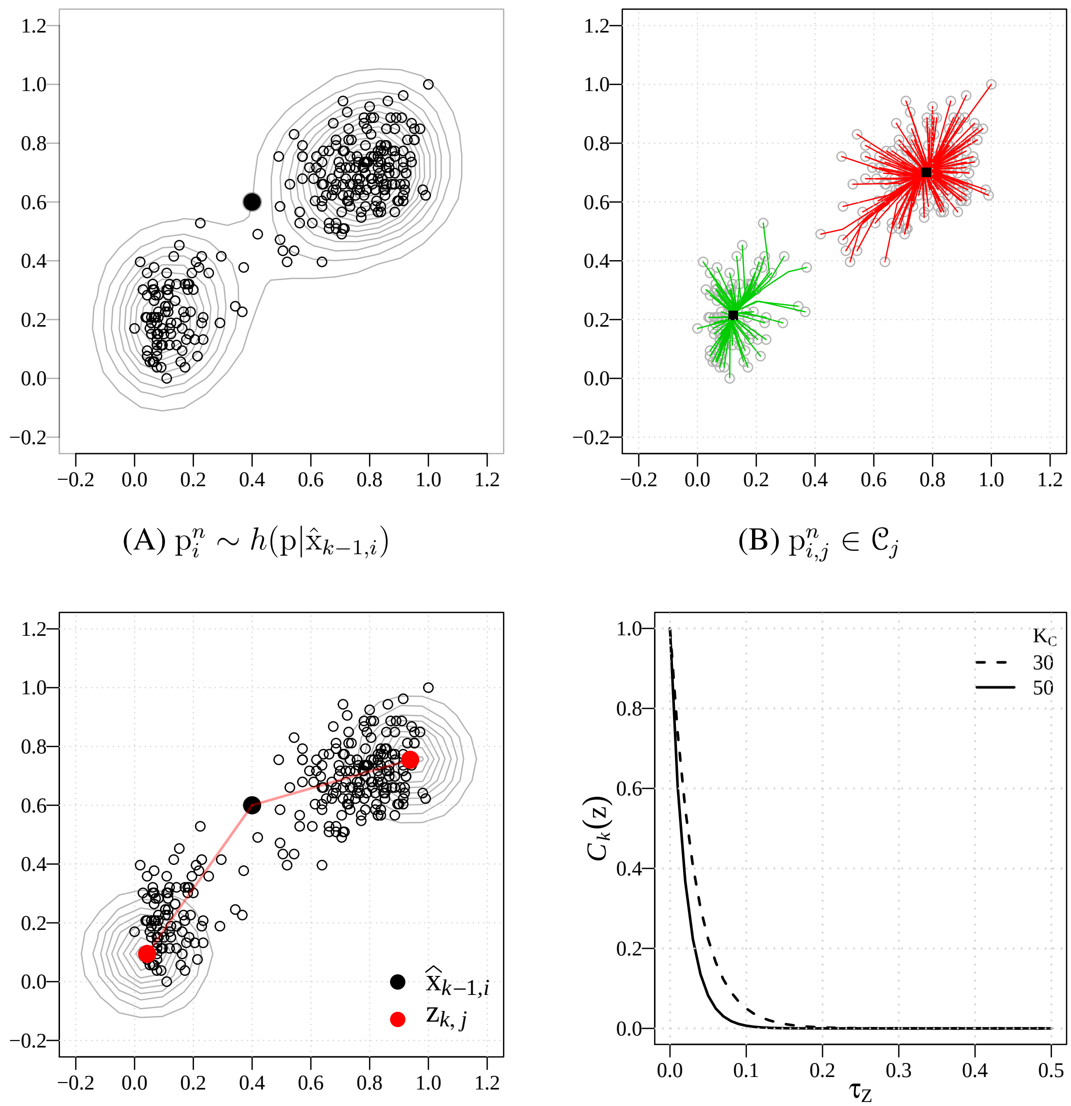


(B) $\beta_{k|k-1}(\mathrm{x}|\mathrm{x}')$



(C) $h(\mathrm{p}|\mathrm{x}')$

Figure S1: Transition densities (2D examples) for persistent (A) and spawned (B) objects with $z = 0$, $\mathrm{x}' = \left[0, 0, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right]$, $\kappa = 2$, and $r_k = 3$. (C) Importance sampling used in the observation model without the tubularity component, $\tau(\mathrm{p}) = 1$, and $\kappa = 0.5$. Rainbow color coding is used running from blue (indicating low values) to red (indicating high values).



(A) $\mathrm{p}_i^n \sim h(\mathrm{p}|\hat{\mathrm{x}}_{k-1,i})$ (B) $\mathrm{p}_{i,j}^n \in \mathcal{C}_j$

(C) $\mathrm{z}_{k,j}$ (D) $C_k(\mathrm{z}) = e^{-K_c \tau_\mathrm{z}}$

Figure S2: Formation of the observations (2D example). (A) For each object i from iteration $k-1$, particles p_i^n are sampled from the importance sampling function h, using the state estimate $\hat{\mathrm{x}}_{k-1,i}$. The solid dot indicates the location of $\hat{\mathrm{x}}_{k-1,i}$ and the contours represent lines of equal particle weight. (B) The particles are processed by mean-shifting resulting in clusters $\mathcal{C}_j$ whose labeled particles are denoted as $\mathrm{p}_{i,j}^n$. (C) Each observation $\mathrm{z}_{k,j}$ is obtained from the representative cluster particle $\mathrm{p}_{i,j}^{\hat{n}}$ as described in the main text. Contours represent lines of equal observation likelihood. (D) The clutter intensity function.

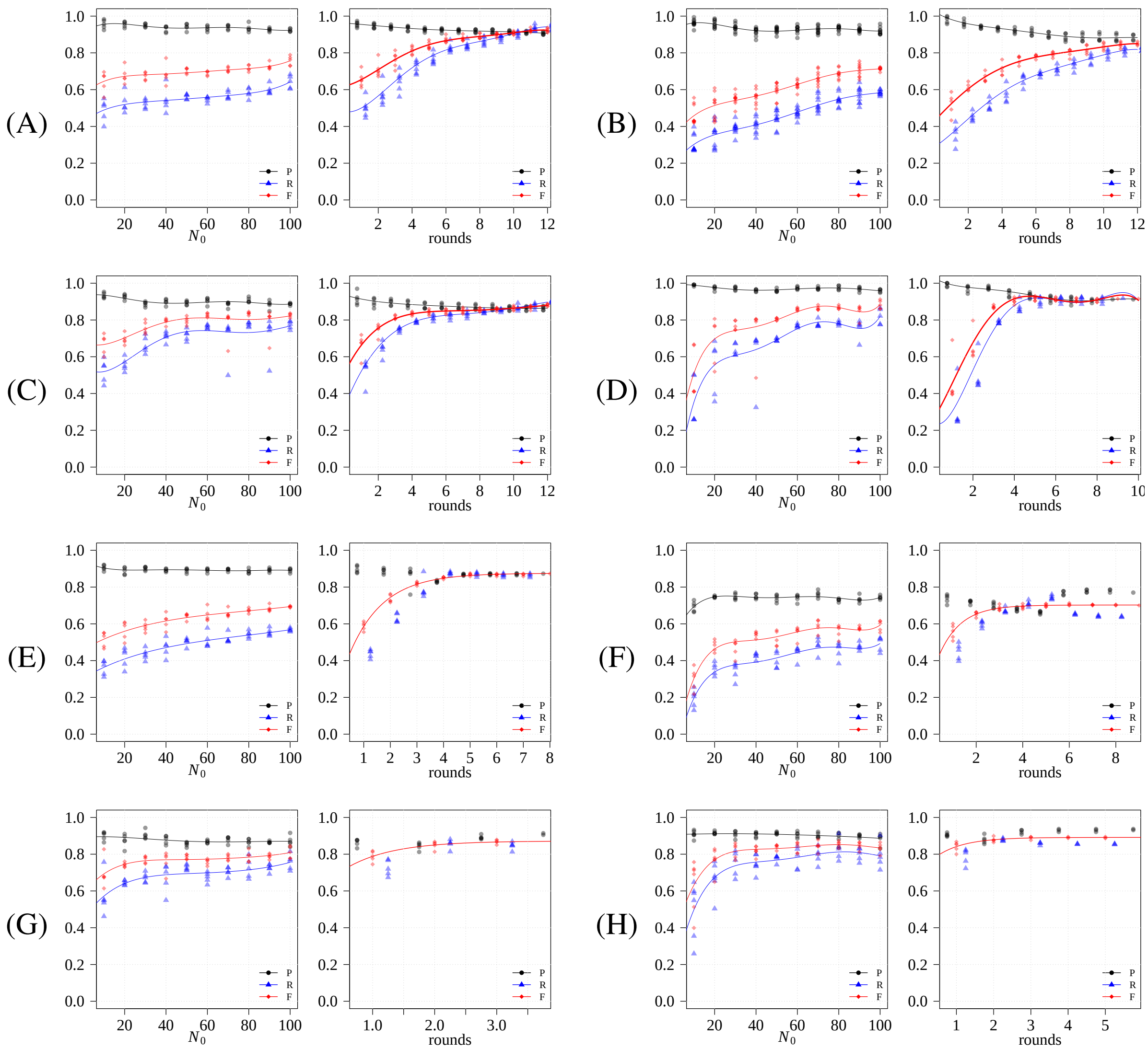


Figure S3: Performance as a function of numbers of seeds and rounds for four example cases from the OPF (A-D) and the HCN (E-H) data set. Similar trends were observed for all cases in the respective data sets. Left panel per case: Precision (P), recall (R), and F-score (F) after one round initialized with different numbers of seeds (N_0). Right panel per case: The scores after multiple rounds with a fixed number of seeds ($N_0 = 40$). Fifth-order polynomial curves were fit to the data to show approximate trends.

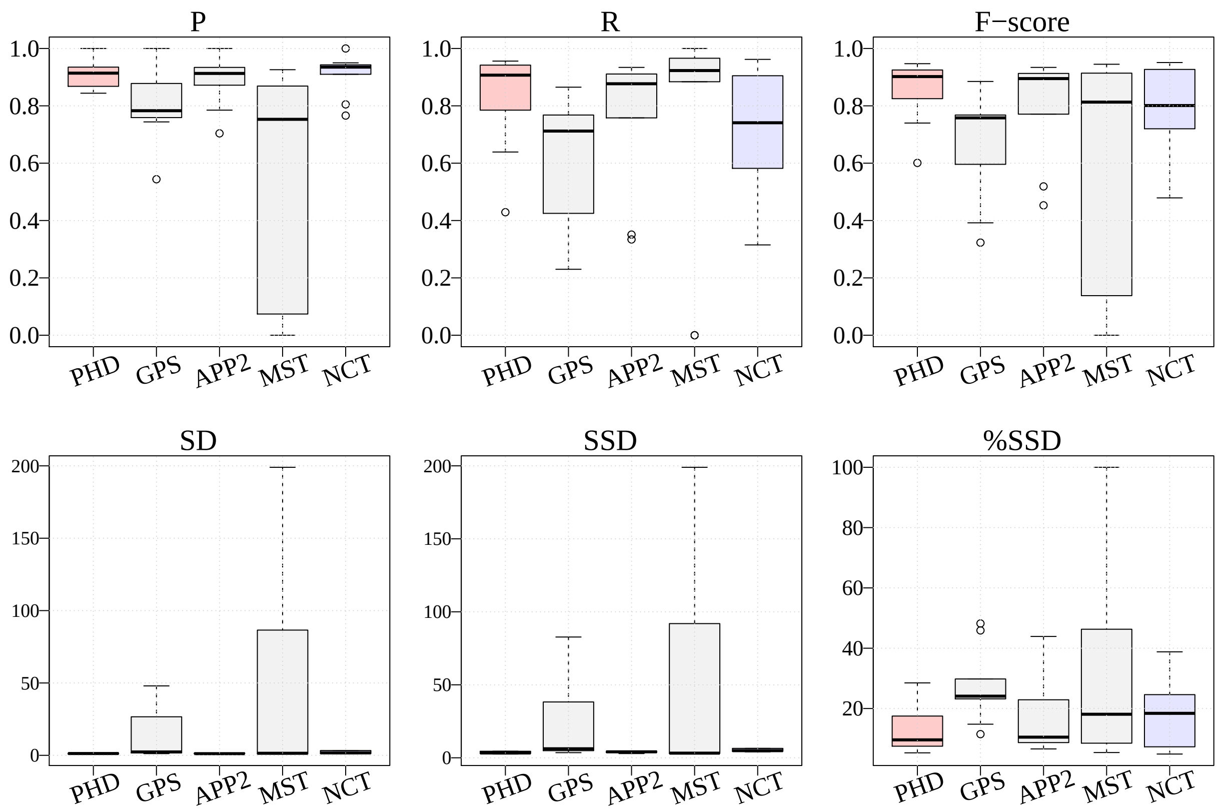


Figure S4: Performance comparison of our method with several other methods on the OPF data set. For each method and each measure, the plotted box indicates the 25-75 percentile, the horizontal bar indicates the median score, and the whiskers and outliers are drawn using the default settings of R.

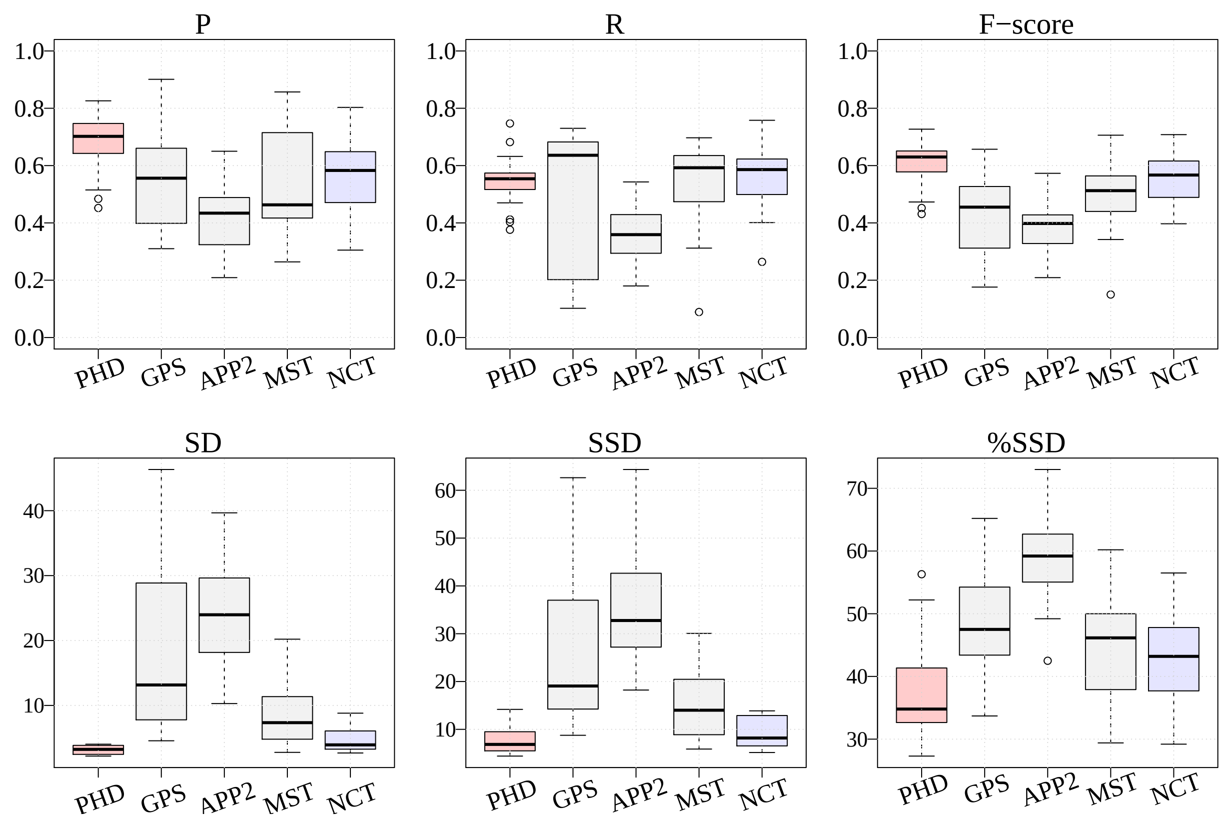


Figure S5: Performance comparison of our method with several other methods on the HCN data set. For each method and each measure, the plotted box indicates the 25-75 percentile, the horizontal bar indicates the median score, and the whiskers and outliers are drawn using the default settings of R.

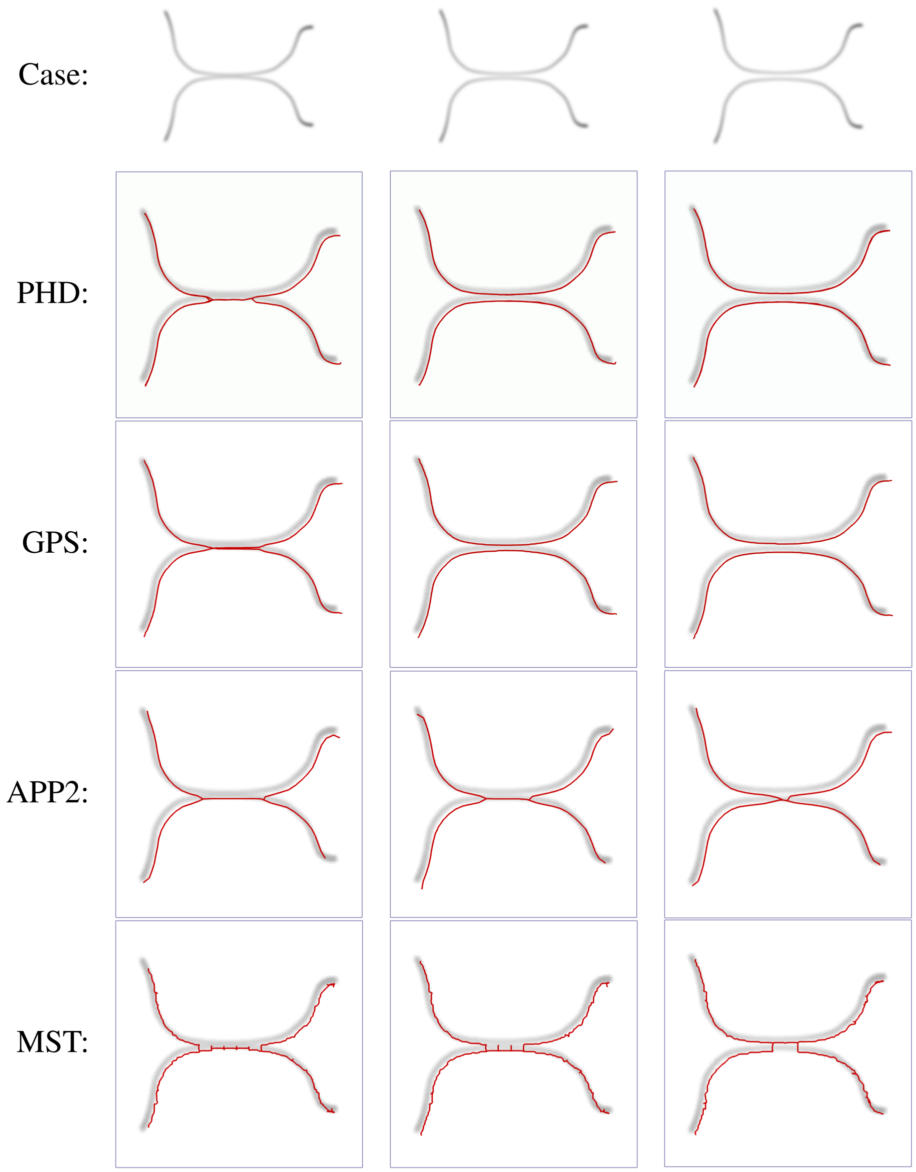


Figure S6: Ability of the tested methods to separate two fibers of similar intensity and scale running closely in parallel. The examples show cases with gradually increasing distance between the fibers: overlap (left column), just separated (middle column), and clearly separated (right column). The tracing results of PHD, GPS, APP2, MST are overlaid (with slight offset) in red color.

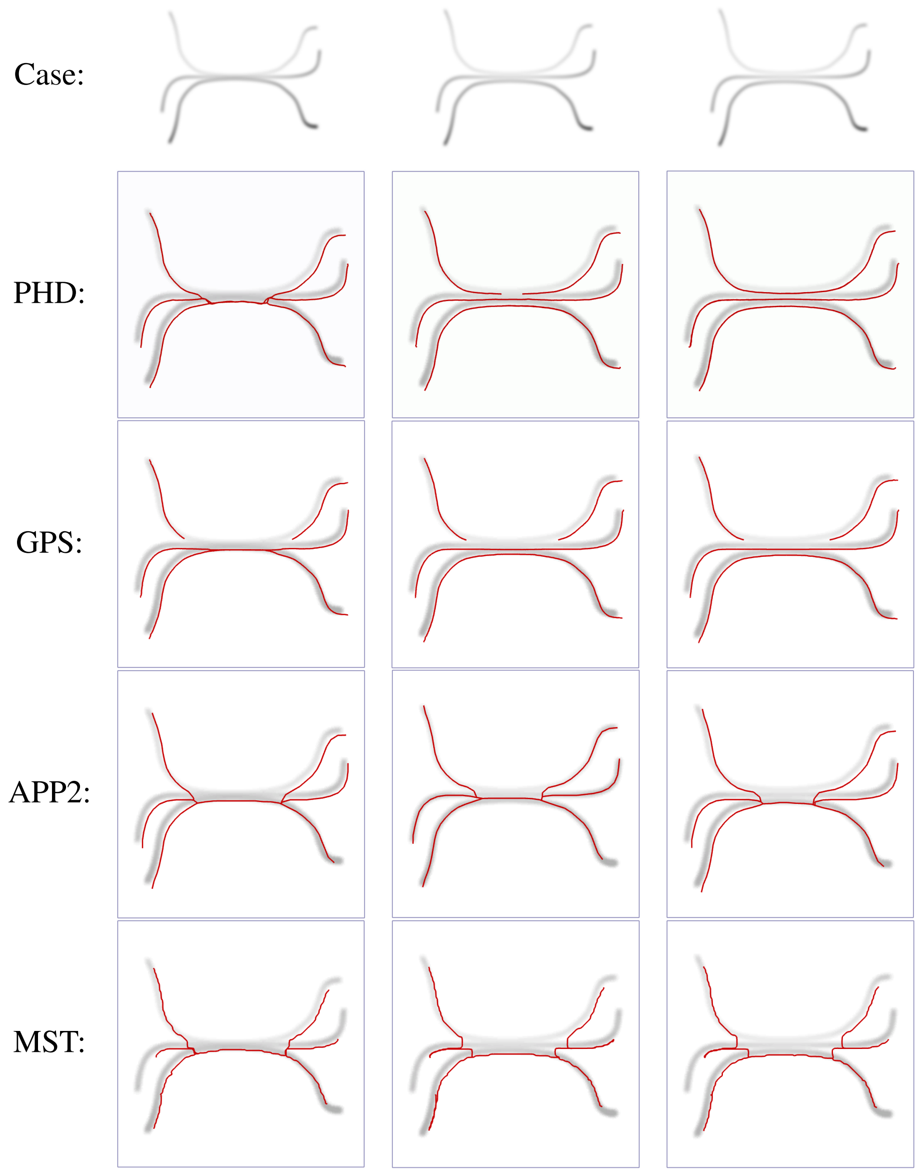


Figure S7: Ability of the tested methods to separate three fibers with different intensity and scale running closely in parallel. The examples show cases with gradually increasing distance between the fibers: overlap (left column), just separated (middle column), and clearly separated (right column). The tracing results of PHD, GPS, APP2, MST are overlaid (with slight offset) in red color.