Underwater Vehicle Localisation using Extended Kalman Filter

Miroslav Radojević¹ and Yvan Petillot²

¹Erasmus Mundus Master in Computer Vision and Robotics (ViBot) student ²Ocean Systems Laboratory, Heriot-Watt University, Edinburgh, UK

Abstract-In order to accomplish various missions, autonomous underwater vehicles (AUVs) need to be capable of estimating their position within the environment. This is a prerequisite of a successful mission since further tasks strongly rely on navigation information. This paper presents the application of an algorithm that would accomplish the localisation of the Ocean Systems Lab's Nessie underwater vehicle using measurements from a number of sensors mounted on it. Well known Extended Kalman Filter (EKF) algorithm approach was suggested as a solution for robot self-localisation. Additional practical issue that was addressed in the work is the choice of heading sensor and quality of the obtained heading as an important ingredient of the navigation. Implementation of the Unscented Kalman Filter (UKF) was investigated as potential improvement in working with nonlinearities. Finally, the absolute position observations tend to be quite noisy but very important measurements for navigation. EKF was demonstrated as a tool for sensor fusion and simultaneous filtering of the position measurements. Experiments with recorded real-time sensor data and real missions have been carried out. Their results have been presented as a part of navigation performance test and analysis.

I. INTRODUCTION

This paper is reporting the application of EKF for localisation of the above mentioned Nessie AUV in an unstructured environment. The concept of sensor fusion was explained. The main contribution is the implementation of an EKF estimator adopted to work on a real underwater vehicle with real-time signals received from sensors. Five degrees of freedom (5DOF) model of the vehicle dynamics was introduced to take the role of the prediction. Work examines the problem from the perspective of engineering a successful AUV navigation in general. The issue of accurate heading and the outliers in absolute position measurement was analysed. Unscented Kalman Filter (UKF) [5] was implemented as an attempt to improve the performance and compensate for the shortcomings of the EKF.

Paper is organised as follows: section § II gives an overview of AUV's navigation capabilities. Section § III introduces the theory of EKF. Section § IV briefly presents each of the measurement devices. Implementation of the localisation module was detailed in section § V. Finally, results are shown in section § VI ending with conclusions and future work in section § VII.



Fig. 1. Standard LBL: A - transponder, B - transducer. Acoustic waves are exchanged between A and B. Detected "time-of-flight" is used to estimate the distance between, hence the position in the network of transponders.

II. NAVIGATION CAPABILITIES OF AUVs

Primary navigation system in most of the applications, including underwater navigation, is Inertial Navigation System (INS) [7]. Motion and rotation information obtained this way are processed in order to provide an estimate of objects location with respect to the initial reference. Since such system accumulates noisy data, it introduces the drift errors that need to be occasionally corrected inside the navigation algorithm. Various ways of correcting those errors were developed. Common "correction tool" is the incorporation of an absolute position measurement in form of GPS (§ IV) or acoustic acoustic based LBL (§ IV) available underwater (Figure 1). Absolute position is inferred from the acoustic feedback of transponders so that the vehicle is capable of locating itself with respect to transponder network. Carrying out underwater vehicle localisation implies introducing concepts such as vehicle state within a navigation strategy framework. Vehicle navigation state describes its position within the environment. Vehicle state is a vector that contains variables relevant for localising the vehicle. In this work, state vector is treated as stochastic - consisted of random variables with Gaussian distribution. As it is the case with random variables, we can say that certain state has an expected value, and that such "randomness" can be expressed with the distribution formula, resulting in descriptor values such as mean and standard deviation that fully describe the distribution in particular case of Gaussian. Most notable stochastic state estimator is Kalman Filter (KF). KF works through iterations by employing the process model for making the state prediction and the observations for doing the state correction [8]. Real

^{*}To whom correspondence should be addressed. Email: miroslav.radojevic@gmail.com

world consists of various nonlinear systems. Practical situations often demand the usage of approximations that eventually lead to linearisation. EKF is a nonlinear version of KF which linearises about the current mean and covariance - hence uses analytic approximations. UKF, on the other hand, is based on sampling [5]. Both treat random variable as Gaussian.

III. EXTENDED KALMAN FILTER

System can be described with set of states that evolve in time according to mathematical functions that are usually nonlinear. Nonlinear state prediction f() would use previous state estimate $\hat{x}(k-1)$, possible control input u(k) and mean value of the process noise (0):

$$\hat{x}(k \mid k-1) = f(\hat{x}(k-1), u(k), 0)$$
 (1)

EKF is intended for solving sub-optimal state estimation of a nonlinear system [4]. The main characteristic of EKF is that it analytically approximates - linearises the process and measurement functions (f() and h()). Linearisation implies approximating these functions with their first derivative around current prediction, similarly as the ordinary math functions are approximated with Taylor polynomials of first degree. In this case, derivation is slightly more complex since model functions f() and h() take several input vectors and output the resulting vector. Hence, the derivation will consist of partial derivation of process per state input vector (Equation 2) and per noise input vector (Equation 3). Partial derivation of measurement function per state (Equation 4) and measurement noise (Equation 5). Partial derivatives themselves will be Jacobian matrices considering that vector is derived per vector.

$$\boldsymbol{F}(k) = \frac{\partial f}{\partial x} (\hat{\boldsymbol{x}}(k \mid k-1), \boldsymbol{u}(k), 0)$$
(2)

$$\boldsymbol{W}(k) = \frac{\partial f}{\partial n}(\hat{\boldsymbol{x}}(k \mid k-1), \boldsymbol{u}(k), 0)$$
(3)

$$\boldsymbol{H}(k) = \frac{\partial h}{\partial x} (\hat{\boldsymbol{x}}(k \mid k-1), 0)$$
(4)

$$\boldsymbol{V}(k) = \frac{\partial h}{\partial m} (\hat{\boldsymbol{x}}(k \mid k-1), 0)$$
 (5)

Subsequently, filtering process can be treated similarly as classic, discrete linear KF (Algorithm 1). Process model mathematically describes how the state changes for the given input (Equation 1). Essential invention in EKF algorithm is the linearisation of the given function around current state mean and variance which further results in estimation process similar to the one described for linear KF [6].

IV. SENSORS

Underwater positioning can utilise different types of sensors combined together in one system. The role of the sensors is to measure absolute position, velocities and heading/orientation. Sensor outputs measure with Algorithm 1 The Discrete Extended Kalman Filter Require: $E\{x(0)\} = x(0) = \hat{x}(0)$ {initialize state} Require: $P(0) = \delta_{jk}P_0$ {initialize covariance}

looh
$k \Leftarrow k + 1$
$\hat{x}(k \mid k-1) = f(\hat{x}(k-1), u(k), 0)$ {state
prediction}
$\mathbf{P}(k \mid k - 1) = \mathbf{F}(k)\mathbf{P}(k - 1)\mathbf{F}^{T}(k) +$
$\boldsymbol{W}(k)\boldsymbol{Q}\boldsymbol{W}^{T}(k)$ {state prediction uncertainty}
$\nu = \boldsymbol{z}(k) - h(\boldsymbol{\hat{x}}(k \mid k-1), 0) \text{ {innovation}}$
$\boldsymbol{S} = \boldsymbol{H}(k)\boldsymbol{P}(k \mid k-1)\boldsymbol{H}^{T}(k) + \boldsymbol{V}(k)\boldsymbol{R}\boldsymbol{V}^{T}(k)$
{innovation uncertainty}
$\mathbf{K} = \mathbf{P}(k \mid k-1)\mathbf{H}^{T}(k)\mathbf{S}^{-1}$ {"Kalman gain"}
$\hat{\boldsymbol{x}}(k) = \hat{\boldsymbol{x}}(k \mid k-1) + \boldsymbol{K}\nu \text{ {state correction}}$
$\boldsymbol{P}(k) = (\boldsymbol{I} - \boldsymbol{K}\boldsymbol{H}(k))\boldsymbol{P}(k \mid k-1)$ {state
correction uncertainty}
return $\hat{\boldsymbol{x}}(k), \boldsymbol{P}(k)$
end loop

reference either in *body frame* (Figure 3(b)), the one fixed to the object or in *global frame* (Figure 3(a)). Basic navigation sensor set for a high-end AUV usually consists of:

Pressure (depth) sensor is standard piece of the equipment for an AUV. Measuring the pressure enables the correlation of the value of pressure with the value of depth. Device can frequently ascertain the absolute depth with good precision, within the range of centimetres.

Magnetic compass provides 3D vector of local magnetic field. It's main role is orientation measurement, particularly heading (yaw). Magnetic compass points at magnetic north. North direction as it appears on maps points to the geographic north ("true north"). That is the direction towards the rotation axis of the Earth. Magnetic declination is an angle between magnetic north (measured by compass) direction and the true north direction (the one that maps refer to). Depending on location where the compass is used, magnetic declination can vary, hence, calibration is necessary. In addition, different magnetic effects can affect the measurement. Compass delivers absolute measurement of heading, prone to noise.

DVL is intended to measure linear velocities. Transceiver components mounted on the device, pointing downwards (towards the bottom) emit acoustic impulses which are expected to be reflected and read. In case reflectance exists, DVL is "bottom-locked" and ready to measure.

FOG is based on measuring the interference of two light beams that pass through a coiled optical fibre in both directions. FOG provides quite precise information on rotation as it delivers the angular information: rate of change of heading (yaw rate).

GPS is a well known satellite-based navigation system that provides position information anywhere on the Earth surface or in the air, reasonably close to the



Fig. 2. Sensor fusion diagram.

surface. Due to absorption of electromagnetic waves in the water GPS signal is not available underwater. Despite the fact that GPS is not available, vehicles are equipped with GPS receiver intended to be used for initial position information before submerging or for occasional position updates if the vehicle temporarily goes back to the surface. Precision of the GPS position information can vary significantly [3]. Such huge deviation can cause significant inaccuracies in navigation.

LBL is an acoustic positioning system which provides the absolute position, a ground-based reference. LBL is used for measuring position with respect to several tethered beacons with known position, placed in water (Figure 1). LBL transceiver "pings" each of the beacons and detects the signal travel time in order to calculate their distance. It can be understood as the extension of the GPS information below the water surface. EKF fuses the measurements from all the devices together: localisation algorithm collects the incoming sensor information and computes the pose of the vehicle by filtering the data cluster obtained from sensor devices. Such procedure is regarded as sensor fusion (Figure 2). Basic sort of sensor fusion implementation is incorporated in navigation algorithm by combining different quantities into a jointly updated state vector with position, orientation and velocities (§ V).

V. IMPLEMENTATION

Position, orientation and velocities of a vehicle underwater are stored within the state vector. Proposed solution for localisation uses state-space approach and EKF to estimate the value of the state vector using data from odometry sensors and acoustic positioning system (LBL), if available. Reasons for choosing this method are influenced by the application itself. Localisation is intended to work in unstructured environments, with no clear visibility, relying on kinetic and absolute position measurements. Mathematical model of the system is the integral part of the EKF. It is used to define the state transition law by applying well known kinematic equations which describe object motion [11]. Constant velocity 5DOF kinematic model is used as system model to predict the movements of the submerged body [9]. States are predicted at each time-step using the model f() and previous state X(k-1) (equation 6).

$$\boldsymbol{X}(k) = f(\boldsymbol{X}(k-1), \boldsymbol{N}(k-1))$$
(6)



(b) AUV body frame - local coordinate system with movement directions.

Fig. 3. AUV state vector values and five degrees of freedom.

Process model is used to describe the state transition in time. In proposed discrete-time stochastic model, 5DOF include position values and two angle states: yaw and pitch - making altogether five possible values to change in modelling vehicle position (figure 3(a)). Since the application uses state-space approach, focus will be on defining a state vector that would incorporate all the relevant values for the dynamic system - kinematic and position variables. In spirit of that, system state vector combines together metric and angular values. At discrete time moment k, it values:

x takes the value of *north* (expressed in meters), yis east and z is depth. a marks the altitude with u, v and w standing for linear velocities: surge, sway and heave velocity, respectfully. The rest of the state vector covers angular values (expressed in radians or degrees). ψ and φ are used as yaw and pitch, hence describing the vehicle orientation. ψ and $\dot{\varphi}$ are angular velocities: yaw rate and pitch rate, respectfully. The state vector incorporates all the relevant information necessary to describe the system under investigation. Angle and velocity for pitch degree of freedom is included in 5DOF system model since it can make a difference in estimating vehicle location in cases of tilted vehicle movement. Model uses previous state and noise to make a prediction on the next state vector value X(k) using non-linear function f() and process noise vector N (Equation 6) where N = \dot{u} \dot{v} \dot{w} $\ddot{\psi}$ $\ddot{\varphi}$]^T. Process noise models inaccuſ racies or unpredictable disturbances in motion model [10]. To summarize, implementing vehicle localization using EKF demands establishing two models: first one describing the state evolution (system model) and the





Fig. 4. Two modes for combining together sensor measurements

into observation.

second model that associates noisy measurement with the state (*measurement model*).

EKF (§ III) was chosen for the state estimation as a logic choice being an algorithm that integrates together different sensor measurements, makes a sub-optimal, recursive state estimation and above all, is derived for nonlinear systems [11]. The main feature of EKF is that it linearises the system model and measurement model nonlinear functions. System model is further developed according to formulas 2, 3, 4 and 5.

Knowing plant model and deriving F(k), W(k) enables EKF algorithm to complete the prediction stage using known formulas. Next step is the correction of the prediction using data obtained from measurement. *Measurement model* introduces measurement equation which establishes the connection between the measurements and the target state (equation 8) where Z(k)represents the measurement at time k, X(k) represents state vector and M(k) represents noise. Purpose of the measurement is to be able to update, correct the state X(k) using measurements Z(k). h() is generally a non-linear function. EKF linearises the measurement model.

$$\boldsymbol{Z}(k) = h(\boldsymbol{X}(k), \boldsymbol{M}(k)) = \boldsymbol{H}\boldsymbol{X}(k \mid k-1) + \boldsymbol{M}(k)$$
(8)

For this particular application and available sensor configuration, state vector elements are measured directly, hence the measurement relation becomes equality. There is no need for partial derivation (Equation 8). Measurement noise is submitted in form of an additive Gaussian zero-mean noise assigned to each measured value. Measurement (observation) noise is characterised with zero mean $(E\{M(k)\} = 0)$ and standard deviation $(E\{M(k)M^T(k)\})$ given as diagonal covariance matrix with diagonal elements set to constant filter parameter σ^2 for each of the measured values (Figure 9). It expresses how much we trust in the measurement, how uncertain or varying measurement of each of the state values is.

One of the features of the process is that measurements are not available all the time. The reason is the nature of the process of estimating the location itself. Simply - messages from sensors arrive at different moments and it happens that some of the sensors could not be available due to different causes (no "dvl lock", for instance). The idea is to take all the available information periodically and integrate it together in measurement model, as a filter observation (Figure 4(b)). Alternatively, each message can be filtered upon its arrival (Figure 4(a)). In case observation is empty, filter does the prediction only. Idea for the solution has been introduced in [9]. Some other implementations of EKF for underwater navigation have reported the usage of similar strategy for merging the measurements together [1], [2]. This way, measurement model adopts to the set of the values observed.

Integrating together several different sensor-specific measurements into one observation would imply concatenating together several measurement matrices (Z, H) and measurement noise matrices (R), as shown in 9 for the sample case of two sensor device measurements included in one observation. EKF maintains its own timer for the observations (value T from the model). As an example, depth sensor measures depth (z) and heave velocity (w), thus

$$\boldsymbol{Z}_{depth} = \begin{bmatrix} z & w \end{bmatrix}^T$$

and

$$m{R}_{depth} = \left[egin{array}{cc} \sigma^2_{depth} & 0 \ 0 & \sigma^2_w \end{array}
ight]$$

$$\boldsymbol{Z}(k) = \begin{bmatrix} \boldsymbol{Z}_{sensorI} \\ \boldsymbol{Z}_{sensorII} \end{bmatrix}, \boldsymbol{H}(k) = \begin{bmatrix} \boldsymbol{H}_{sensorI} \\ \boldsymbol{H}_{sensorII} \end{bmatrix}, \boldsymbol{R}(k) = \begin{bmatrix} \boldsymbol{R}_{sensorI} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_{sensorII} \end{bmatrix}$$

. Where σ marks the standard deviation expressing how much we trust in measurement. It is given as EKF parameter. Similar pattern values for the other sensors depending on values that they measure. Having more than one measurement involved in estimation of the global state is a good characteristic. The estimate which uses more diverse data gives better estimate since it is possible to combine together more that one sort of observation. Another advantage follows the fact that the whole set of state variables is updated each time, resulting in more correlation between variables. Hence those that are missing for some reason can be compensated this way. Results of simulations using authentic data and the real missions are given in Section § VI. Odometry integrates velocity and acceleration data collected from devices such as gyroscope or accelerometers. Integration of noisy data over time or usage of "relative measurements" (those calculated from absolute measurements) results in drift or bias of the final estimate. In order to recover from that, algorithms perform the correction. Correction takes an absolute measurement which should be less precise, possibly noisy, but not prone to drifting.

VI. RESULTS

Nessie missions were carried out as a part of the algorithm trials. It is useful to mention that there is no exact ground truth for underwater robot localization available. GPS signal, if available, could serve as an absolute position reference: either directly or in form of LBL. Experimental results have been obtained for different missions. Good news, however, is that the absolute depth measurement is quite accurate and frequent, making AUV localisation a 2D task. For a high-end underwater vehicle such as Nessie, main source of navigation error is influenced by transformation of vehicle-referenced velocities to world-referenced velocities, particularly due to yaw (heading) measurement errors. Yaw can be measured directly using magnetic compass or integrating FOGbased yaw rates. Simulation with data from previous missions was carried out to see which device gives the best performance for a given underwater vehicle and possibilities of improvement using sensor fusion. Dead reckoning localisation substituted with occasional LBL position updates was compared with the localisation obtained after filtering (Figure 5) for the recorded straight line trajectory mission.

Heading calculated by integrating FOG's yaw rate - tends to be accurate and fast, less prone to noise. Nevertheless, it is calculated each time by appending yaw rate value integrated in time on the previous yaw value (relative measurement). Therefore, it is sensible on initial absolute heading measurement. In case initial yaw is imprecise, a constant bias exists in



(a) N/E localisation. High confidence in yaw measurement.



(b) N/E localisation. Confidence in yaw measurement lowered.



(c) Heading estimation. Biased yaw measurement is being corrected by decreasing confidence in yaw measurement, SDyaw = $0.2rad \approx 11.5^{\circ}$. SDyawRate = $0.004 \frac{rad}{s}$.

Fig. 5. AUV localisation using EKF. Yaw was measured by integrating yaw rate periodically measured using FOG.

yaw measurement (Figure 5(a)). Thus, putting high trust in yaw measurement is not a recommendable strategy. Eventually, after assigning less confidence in yaw measurement (higher σ_{yaw} value), bias becomes filtered out if measured and filtered heading are compared (Figure 5(c)). Tests show that localisation performance can be tailored by setting the confidence in prediction model or measurement values. Confidence is materialized as the variance of the random variable: the lower it is, more certain the value of the random variable is hence more confident in value of that variable we tend to be. EKF tries to optimise the



Fig. 6. Spiral trajectory and the trajectory estimation using EKF.

result within the defined boundaries of uncertainty.

Trajectory filtering is shown in example of spiral trajectory and surfacing action that was taken with Nessie starting from the depth of around 12 m. EKF estimation results are shown in Figure 6 together with LBL position updates and dead reckoning starting from each position. Similarly as with previous plots LBL-aided-dead reckoning was shown together with LBL position updates. Filtered trajectory does not experience severe jumps, and the curve seems to be smoother and less prone to drifting. Standard deviation of north and east measurement parameters was tested with different values, causing more or less confidence in LBL measurement hence shaping the localisation curve and managing filtering of the LBL outliers.

Rectangular trajectories were tested in low depths of a lake, with the GPS signal available to be used as a position reference and ground truth indication (Figure 7). Dead reckoning navigation was used as a reference when controlling the vehicle movement during the experiment. This fact can cause slight confusion in analysis of the trajectory graphs since all the dynamics and forces were applied with respect to the dead reckoning navigation which is an estimated value. GPS signal available from the antenna located on the water surface is serving as a measure of absolute position within the lake - giving an idea about the actual vehicle position while it tries to moves within the boundaries of estimated dead reckoning position. Main issue when performing the square trajectory tests was significant imprecision of GPS signal (Figure 7(a)). GPS measurements were appended to the observations. EKF and UKF localisation results were shown in



(a) N/E localisation of a rectangular trajectory ($\sigma_{north} = \sigma_{east} = 0.5m$).



(b) Filtered absolute position, linear velocities and heading recorded while making a rectangular trajectory.

Fig. 7. GPS aided rectangular trajectory localisation using EKF and UKF.

Figure 7(a) for $\sigma_{north} = \sigma_{east} = 0.5m$. UKF was compared with the EKF localisation, under same parameter settings and using the real data obtained from Nessie sensors. Trajectory obtained using UKF tends to be slightly more precise compared with the one obtained using EKF. Since it is more precise algorithm when approximating nonlinearity [5], UKF preserves the formula of prediction model better. EKF tends to filter the GPS-measured north-east (N/E) coordinates, hence partially corrects the GPS imprecisions. At this point it is evident why EKF is a great tool. Filter tries to satisfy the set uncertainty boundaries and fuse all the available information trying to make the most out of it combined together in one mathematical system. Furthermore, fusing such imprecise and sketchy position data from GPS, still improves the localisation. Obtained trajectory tends to go towards what can be treated as expected path. From something that looked like a noisy collection of position observations at the beginning (Figure 7(a)), application of EKF together with sensor fusion enabled having generally better navigation performance.

VII. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

The main focus of the work presented is practical application of Extended Kalman Filter for Ocean System Lab's Nessie AUV navigation module. EKF was designed to estimate the location of an underwater robot by processing real-time inertial and position information obtained from sensors. Furthermore, EKF algorithm was utilized as a framework for accomplishing sensor fusion - blending together measurements from different sensors as a part of the estimation process. The issues that were addressed in the thesis include suitable management of measurement tasks among mounted sensor devices and the role of EKF in correcting deficiencies. Specific case of heading measurement was tested, since this type of angular information is particularly important for the navigation. In conclusion, EKF proves to be useful navigation tool with several convenient features: capable of successfully combining together different sensory information into a location estimate that tends to be optimal with respect to set expectations, or recovering from the missing measurements, corrupted position information, outliers, or signal noise. Implementation of UKF for localisation would improve the accuracy of approximating nonlinearities in EKF at the same computational cost.

B. Future Works

Future work on improving localisation performance involves more trials with the vehicle trajectory fixed to known landmarks, so that the results of localisation could be thoroughly evaluated with trustful ground truth. Experiments that involve tilted vehicle movements could make an evaluation of the influence of the 5th DOF on the quality of localisation. EKF could be improved so that it works with control inputs - which could contribute in robustness of the localisation. Finally, the problem of correcting the absolute position with LBL information gives space for improvement since the measured position tends to be quite uncertain and prone to different sorts of noise. Solution for rejecting outliers could rely on some version of back-filtering - filtering based on history of received observations.

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